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Combining Space-Time Block Modulation with Integer Forcing Receivers

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***Abstract*—In a recent work by Zhan et al., a new equalization technique for multiple-input multiple-output channels, applicable when linear codes are used for transmission, was proposed. In this technique, coined integer-forcing equalization, the channel matrix is equalized at the receiver to one consisting of only integer entries. It was demonstrated that for quasi-static independent flat Rayleigh fading, and when independent streams are sent over each transmit antenna, this equalization technique allows to approach quite closely maximum-likelihood performance, using standard coding and decoding of off-the-shelf linear codes designed for transmission over a scalar white Gaussian channel. In particular, the technique is optimal in the diversity-multiplexing tradeoff sense. In the present work, we describe how this receiver structure may be seamlessly combined with linear transmit diversity methods. We show that for quasi-static independent flat Rayleigh fading, this combination allows to approach closely the outage capacity of the channel using linear pre- and post-processing and standard scalar coding and decoding.**

I. Introduction

It is well known that increasing the number of antennas in a wireless system can significantly increase throughput. In an open-loop setting, this throughput gain comes however at the price of substantially increased receiver complexity. A great deal of work has thus gone into designing reduced complexity multiple-input multiple-output (MIMO) architectures that allow to trade between complexity and performance. In this work, we describe an architecture that allows to closely approach the optimal theoretical performance, at least when the channel model is that of independent Rayleigh fading, while utilizing only linear pre- and post-processing in conjunction with scalar coding and decoding of standard linear codes designed for the scalar additive white Gaussian noise (AWGN) channel.

We consider a MIMO system, where a transmitter equipped with N_t antennas communicates with a receiver equipped with N_r antennas. We further consider an open-loop mode of operation, i.e., the transmitter is assumed to have no knowledge of the channel. The receiver is assumed to have perfect channel state information.

The complex baseband received signal at time t is

$$\boldsymbol{y}_t^{\mathrm{rec}} \quad = \quad \mathbf{H}\boldsymbol{x}_t + \boldsymbol{n}_t^{\mathrm{rec}} \tag{1}$$

where $\boldsymbol{x}_t = [x_{1,t}, ..., x_{N_t,t}]^T$ is the vector of transmitted symbols at time t, $\boldsymbol{y}_t = [y_{1,t}, ..., y_{N_r,t}]^T$ is the vector of received symbols at time t and $\boldsymbol{n}_t^{\mathrm{rec}}$ is independent identically distributed (i.i.d.) circularly-symmetric complex white Gaussian noise with power N_0.

We can equivalently rewrite (1) over the reals as

$$\begin{bmatrix}\mathrm{Re}(\boldsymbol{y}_t^{\mathrm{rec}})\\ \mathrm{Im}(\boldsymbol{y}_t^{\mathrm{rec}})\end{bmatrix} = \begin{bmatrix}\mathrm{Re}(\mathbf{H}) & -\mathrm{Im}(\mathbf{H})\\ \mathrm{Im}(\mathbf{H}) & \mathrm{Re}(\mathbf{H})\end{bmatrix}\begin{bmatrix}\mathrm{Re}(\boldsymbol{x}_t)\\ \mathrm{Im}(\boldsymbol{x}_t)\end{bmatrix} + \begin{bmatrix}\mathrm{Re}(\boldsymbol{n}_t^{\mathrm{rec}})\\ \mathrm{Im}(\boldsymbol{n}_t^{\mathrm{rec}})\end{bmatrix}. \tag{2}$$

We note that the architecture we discuss is general and can be used over channel of arbitrary statistics. Nonetheless, in our performance analysis we assume a quasi-static independent flat Rayleigh fading model. As a benchmark for the optimal performance of open-loop MIMO we use the mutual information of isotropic Gaussian transmission.

$$I_{\mathrm{OPT,OL}} = \log_2 \left| \mathbf{I}_{N_r} + \frac{\mathrm{SNR}}{N_t}\mathbf{H}\mathbf{H}^H \right| \tag{3}$$

where SNR is defined as

$$\mathrm{SNR} = \frac{\varepsilon_s}{N_0},$$

and ε_s is the total transmit power. In the sequel we refer to $I_{\mathrm{OPT,OL}}$ as the white-input (WI) mutual information. Thus, the WI mutual information corresponds to an i.i.d Gaussian codebook (in time and space) in conjunction with maximum-likelihood decoding.[1]

We assume that the channel is constant throughout the duration of the code block (i.e., a quasi-static model) and we do not perform coding over multiple channel realizations. In such a setting one may consider the maximal supported rate at a given outage probability. The outage probability for a given transmission rate (assuming very long block length) is defined as the probability that the channel does not support the chosen target rate R, i.e.,

$$P_{\mathrm{outage}}(\mathrm{SNR}) \triangleq \Pr\left[\log_2 \left| \mathbf{I}_{N_r} + \frac{\mathrm{SNR}}{N_t}\mathbf{H}\mathbf{H}^H \right| < R(\mathrm{SNR})\right]. \tag{4}$$

[1] We note that according to Telatar's conjecture, optimal transmission in terms of minimizing the outage probability, requires WI transmission from possibly only a subset of antennas. For high target outage probabilities, one may interpret N_t to be this number rather than the total number of antennas.

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In an open-loop mode of operation, one of the most important properties of a system is the diversity it offers. The diversity is defined as the slope (at high SNR) of the error probability as a function of SNR (in a logarithmic scale). The higher the diversity, the more robust the system is. Under the Rayleigh channel model considered, the maximal achievable diversity order is equal to $N_t \times N_r$.

One design goal is to attain this slope when working at a fixed rate. Another design goal may be to achieve a rate that scales according to the maximal possible multiplexing gain (i.e., $\min(N_t, N_r)$) as a function of SNR. It is well known that these two figures of merits, diversity gain and multiplexing gain, may be viewed as a "linked" resource whereby one can trade one for the other. A very useful framework for this tradeoff is given by the diversity-multiplexing tradeoff (DMT) as defined in [1].

While achieving the optimal DMT is a necessary condition for a scheme to operate "close" the optimal possible performance, it is not a sufficient one as the DMT is only a coarse characterization. Specifically, achieving the DMT does not imply a small (or even bounded, as the SNR increases) gap-to-WI mutual information. As our goal is to approach the latter, as a first step we restrict attention to linear modulation schemes that achieve the DMT.

A. DMT optimal uncoded schemes

For the case of two transmit antennas and a single receive antenna, Alamouti [2] showed that by applying linear pre- and post- processing, the 1×2 MISO channel can be converted to a scalar channel having mutual information equal to the WI mutual information of the MISO channel. It follows thus (see, e.g., [1]) that in this case, uncoded transmission in conjunction with Alamouti modulation suffices to achieve the optimal DMT since this is the case for scalar channels. Furthermore, standard scalar AWGN code can be used to approach the optimal performance at high SNR. That is, over 1×2 MISO channel, one may approach the optimal possible performance to the same extent and with roughly equal complexity as for a scalar AWGN channel.

An important extension of Alamouti modulation is linear dispersion "codes". In the sequel, we refer to this method as linear *modulation*. This modulation method spreads the data symbols across space (different antennas) and (T) channel uses. Thus, we can write the resulting transmission matrix as

$$\tilde{\mathbf{X}}_k = \mathbf{P}\boldsymbol{x}_k \tag{5}$$

where $\boldsymbol{x}_k$ is an $N_t \cdot T \times 1$ vector consisting of $N_t \cdot T$ symbols to be sent in the k'th modulation block (in case of full-rate linear dispersion modulation, all these symbols are different, non-zero symbols), $\tilde{\mathbf{X}}_{i,j}$ is the symbol transmitted by antenna i at time j, that belongs to the k'th modulation block, and $\mathbf{P}$ is a $N_t \cdot T \times N_t \cdot T$ linear modulation matrix. A detailed definition and analysis can be found in [3]. Using linear dispersion modulation (1) can be rewritten as

$$\mathbf{Y}^{\text{rec}} \;=\; \mathbf{H}\tilde{\mathbf{X}} + \mathbf{N} \tag{6}$$

where $\mathbf{Y}_{i,j}$ and $\mathbf{N}_{i,j}$ are the received symbol and noise, respectively, at antenna i at time j, where for notational simplicity we drop which omit the block index k.

Denoting $\mathbf{Y}^{\text{rec}} = \begin{bmatrix}\boldsymbol{y}_1 & \boldsymbol{y}_2 \cdots \boldsymbol{y}_T\end{bmatrix}$, we define $\boldsymbol{y}_{\text{vec}} = \text{vec}(\mathbf{Y}^{\text{rec}}) = \begin{bmatrix}\boldsymbol{y}_1 & \boldsymbol{y}_2 & \ldots & \boldsymbol{y}_T\end{bmatrix}^T$. We may rewrite (6) as

$$\boldsymbol{y}_{\text{vec}} \;=\; \mathcal{H}\tilde{\boldsymbol{x}}_{\text{vec}} + \boldsymbol{n}_{\text{vec}}. \tag{7}$$

The resulting equivalent channel has the following (block-diagonal) structure

$$\mathcal{H} = \begin{bmatrix} \mathbf{H} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{H} & \mathbf{0} & \ldots & \mathbf{0} \\ & & \vdots & \ddots & \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} & \mathbf{H} \end{bmatrix}. \tag{8}$$

A family of linear modulations which achieves the optimal DMT with minimal $T = N_t$ is that of "perfect codes" [4]. An example of such modulation is the golden code [5], [6] for the case of two transmit antennas. In conjunction with uncoded transmission, it achieves the optimal DMT (see [5]). The explicit linear modulation in this case (over the reals) is

$$\mathbf{P} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & 1 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & 0 & \frac{3}{\sqrt{10}} & -\frac{1}{\sqrt{10}} & -\frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} & \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & 0 & 0 \\ 0 & 0 & \frac{3}{\sqrt{10}} & -\frac{1}{\sqrt{10}} & \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} & -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 & 0 & 0 & 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}. \tag{9}$$

We note that in the common terminology used (see, e.g., [7]), oftentimes no distinction between coding and modulation is drawn. Thus, for instance, the block structure of the channel (8) is not utilized for complexity reduction but rather the received vector is regarded (when linear codes are used for transmission) as a noise corrupted point of a space-time lattice. In this paper the distinction between coding and modulation plays a crucial role in the decoding process. Rather than coping with the linear transformation performed by the channel as part of the decoding process, the channel is linearly equalized. More specifically, at the transmitter, the data is encoded by N_t standard linear AWGN encoders (i.e., a turbo, LDPC, or convolutional code), the output of which are linearly transformed (modulated) using the linear dispersion matrix. At the receiver, an integer-forcing equalizer (as recalled next) is applied, the output of which is passed to the corresponding scalar AWGN decoders. See Figure 2.

We further note that some works consider the concatenation of a linear space-time "code" with an outer code. However, regarding the linear space-time modulation as a code leads to a very different receiver architecture than that proposed in this work. See, e.g., Chapter 5 of [8]. Namely, every block of space-time modulated symbols is first detected (either with hard or soft decisions), and these decisions are passed to the outer codes, possibly with iteration. In the proposed architecture, the effect of the linear modulation is absorbed into the channel, the resulting effective channel is equalized, and no detection is done prior to applying standard AWGN decoders.

B. Integer-forcing equalization

In [9], a receiver architecture scheme coined "integer forcing" was proposed. It was assumed the information bits are fed into N_t standard AWGN linear encoders [2] which produce N_t channel inputs (for example, $\boldsymbol{x}_m$ for the m'th antenna). At the receiver, a *linear equalization* matrix $\mathbf{B}_{\rm INT}$ is applied, where $\mathbf{B}_{\rm INT}$ is designed such that the resulting equivalent channel $\mathbf{A} = \mathbf{H}\mathbf{B}_{\rm INT}$ is a matrix whose entries are all integers. This ensures that the output of the channel (without noise) is a valid codeword. The output of the equalizer is next passed to standard (up to the additional element of a modulo operation, which in this paper we assume is a one-dimensional modulo operation, as mentioned in footnote 2) AWGN decoders. Finally, the operation $\mathbf{A}^{-1}$ (again, modulo arithmetic is assumed) is applied. The scheme is depicted in Fig. 1. We note that IF equalization may also be combined with successive interference cancellation (SIC) [11], resulting in improved performance. See [9] for further details.

The main limitation of the scheme presented in [9] is that transmit diversity was not utilized. It has been shown that the scheme, beyond achieving full receive diversity, offers good performance in terms of the gap from the performance of ML detection (of the independent streams transmitted). Our goal in the present work is to allow us to approach the WI benchmark (3), for which transmit diversity must be incorporated.

II. Combining Linear Space Time Modulation with Integer Forcing

A natural extension for the equalization scheme of [9] is to combine it with a modulation scheme that adds transmitter diversity. It is beneficial to choose a modulation scheme that does not require any change in the receiver architecture. This will be the case as long as we restrict attention to linear transmit diversity techniques. Thus, linear modulation results in a new equivalent channel, to which IF equalization may be applied. It seems reasonable to choose as candidate modulations, only those that achieve the optimal DMT for uncoded transmission (and with ML detection). A high level diagram of the considered architecture is depicted in Fig. 2.

[2]We note that standard linear codes do not incorporate shaping and thus may lose up to 0.254 bit per real degree of freedom w.r.t. the expressions derived in this paper, in addition to the gap-to-capacity of these codes when operating over an AWGN channel. See, e.g., [10].

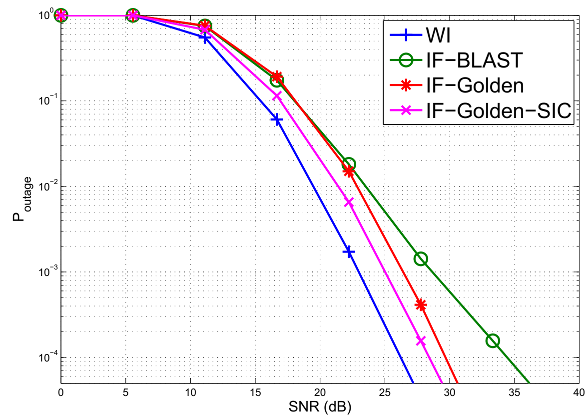

Fig. 3. Comparison of outage probability of different schemes for a target rate of 8 bits per channel use when operating over a 2×2 quasi-static independent flat Rayleigh fading channel.

Combining linear modulation with IF equalization may be viewed as applying IF to the following equivalent MIMO channel

$$\boldsymbol{y}_{\rm vec} \quad = \quad \tilde{\mathcal{H}}\boldsymbol{x}_{\rm vec} + \boldsymbol{n}_{\rm vec}, \tag{10}$$

where $\tilde{\mathcal{H}} = \mathcal{H}\mathbf{P}$. The equalization matrix $\mathbf{B}_{\rm INT}$ and the integer matrix $\mathbf{A}$ are obtained exactly as in [9], substituting $\mathbf{H}$ with $\tilde{\mathcal{H}}$.

We note that finding the IF equalizer requires roughly the same complexity as detection of a single modulation block of $N_t \times T$ uncoded symbols. The advantage of the proposed approach (when a code is used, as considered in this work) is that the equalizer must be computed only once throughout the whole transmission block.

We demonstrate the performance of the proposed architecture via a simulation over a 2×2 MIMO channel with Rayleigh statistics. Figure 3 depicts the outage probability for different transmission schemes with a target rate of 8 bits per channel use. We assume here that the scalar AWGN code is capacity achieving. As mentioned in footnote 2, an additional loss of up to 0.254 bit per real symbol (or 1.53 dB at high SNR) may be inflicted in addition to the gap-to-capacity of the code over an AWGN channel.

In Figure 3 "WI" represents the performance of an i.i.d. Gaussian code with ML decoding; the "IF-BLAST" curve shows the performance of using IF equalization when independent (yet coded) streams are transmitted as in [9]; the "IF-Golden" curve corresponds to the performance of golden-"code" modulation with IF equalization and "IF-Golden-SIC" corresponds to the same but with IF incorporating SIC. It is evident form the figure that the performance of the proposed architecture is quite close to WI performance benchmark. For instance, at an outage probability of 10^{-4}, the the gap of "IF-Golden" is $\sim$ 3.7 dB and that of "IF-Golden-SIC" is $\sim$ 2.3 dB.

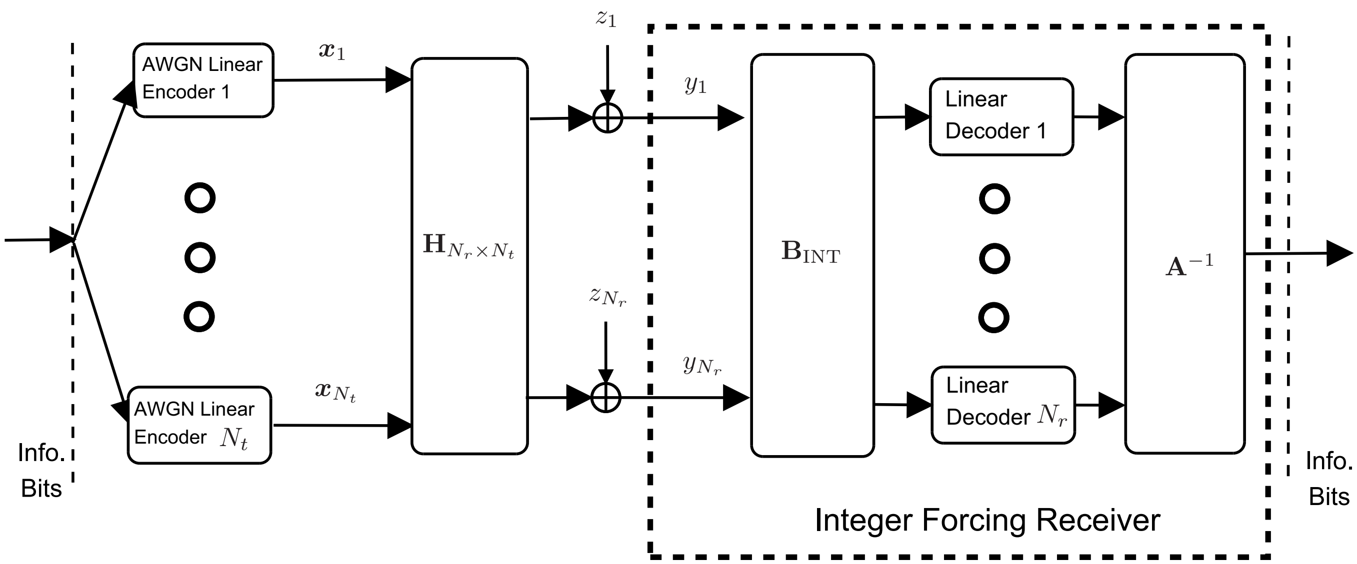

Fig. 1. Integer-forcing equalization.

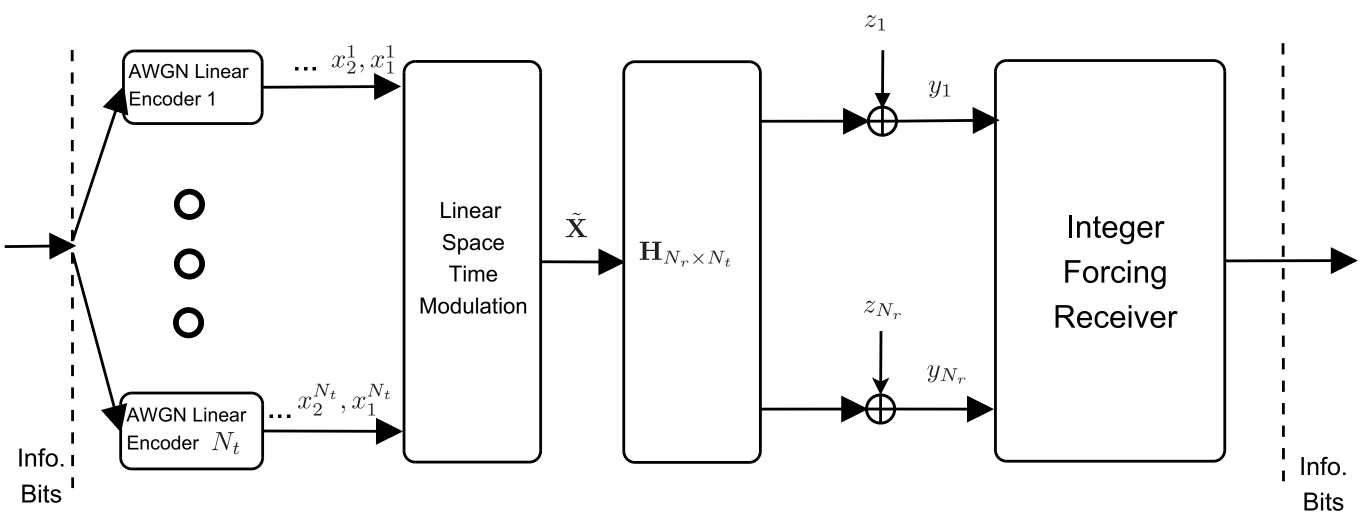

Fig. 2. Combining linear modulation with with integer-forcing equalization.

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